

15 Integralrechnung

15.1 Unbestimmte Integrale

1. a) $2x^3 + 4x^2 - 3x + C$; b) $2 \ln |x| + C$; c) $3 \cdot \frac{3}{4} x^{\frac{4}{3}} + C$;
d) $0,2e^{-x} + C$; e) $0,5(e^x + e^{-x}) + C$.
2. a) $\frac{1}{16}x^4 \ln x - \frac{1}{64}x^4 + C$; b) $e^x x^2 - 2(e^x x - e^x) + C$;
c) $x \left((\ln x)^2 - 2(\ln x - 1) \right) + C = x \left((\ln x)^2 - 2 \ln x + 2 \right) + C$.
3. $\ln |x^3 - 4| + C$.

4. $x^2 + 4 = t^2 \Leftrightarrow x = \sqrt{t^2 - 4}$
 $2x \, dx = 2t \, dt \Leftrightarrow dx = \frac{2t}{2x} \, dt = \frac{t \, dt}{\sqrt{t^2 - 4}}$
$$\int x \sqrt{x^2 + 4} \, dx \Rightarrow \int t \sqrt{t^2 - 4} \frac{t \, dt}{\sqrt{t^2 - 4}} = \frac{1}{3} t^3 + C$$
$$= \frac{1}{3} (x^2 + 4)^{\frac{3}{2}} + C .$$

15.2 Bestimmte Integrale

1. a) $\int_1^4 (3x^2 - 4x) \, dx = [x^3 - 2x^2]_1^4 = (4^3 - 2 \cdot 16) - (1^3 - 2 \cdot 1)$
 $= 64 - 32 - 1 + 2 = 33$;

b) $\int_{-1}^2 \left(10x^4 - \frac{1}{x^2} \right) \, dx = \left[2x^5 + \frac{1}{x} \right]_{-1}^2$
 $= \left(2 \cdot 2^5 + \frac{1}{2} \right) - \left(2(-1)^5 + \frac{1}{-1} \right)$
 $= 64 + \frac{1}{2} + 2 + 1 = 67,5$.

2. $[e^x (x - 1)]_0^1 = +1$.

3. $\int_0^{e-1} \frac{e - 1 - x}{1 + x} \, dx = \int_0^{e-1} \left(\frac{e}{1+x} - \frac{1+x}{1+x} \right) \, dx$
 $= \int_0^{e-1} \frac{e}{1+x} \, dx - \int_0^{e-1} 1 \, dx$
 $= [e \ln(1+x)]_0^{e-1} - [x]_0^{e-1}$
 $= e - 0 - e + 1 - 0 = 1$.

4. $\int_{-1}^1 (4x^3 + 3x^2 + 4) \, dx = [x^4 + x^3 + 4x]_{-1}^1 = 10$.